grasp the basics of band theory **Band Theory** FOR DUMMIES

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Discover how easy it is to

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Solid materials = Periodic solids

The electronic structure of solids (1D, 2D or 3D) is usually discussed in terms of band theory

• The "molecular" approach: Linear H_n oligomers (n = 1, N ~ 10²³)



• The "solid state" approach: Infinite chain of equidistant H atoms



where: k = index (wave vector, vector belonging to the reciprocal lattice) $R_j = ja$ (j, integer varying from $-\infty$ to $+\infty$) $\chi_j = 1s(H_j)$



 $\mathsf{E}(k) = \mathsf{f}(k)$

Periodical function in the reciprocal space (period = $2\pi/a$) $\phi(k)$ and $\phi(-k)$ are degenerate, i.e. E(k) = f(k) is a centrosymmetrical function

Thus, the study of the function E(k) = f(k) can be restricted to the interval: $0 \le k \le \pi/a$ (Irreducible part of the first Brillouin zone)





 $\phi(k) \longrightarrow E(k)$ and $\phi(-k) \longrightarrow E(k)$ Band structure: E(k) = f(k) for $0 \le k \le \pi/a$





Band Width

Dispersive vs flat bands

The band width *W* depends on the overlap between neighboring atoms, *i.e.* on the interatomic distance *a*.



Density of States (DOS)



 $n(E) \alpha (\delta E(k)/\delta k)^{-1}$

NB: The area of the surface delimited by the curve DOS(E) = f(E) is constant when a varies

The Fermi Level (ε_F)



The Crystal Orbital Overlap Population (COOP) curve

Mulliken overlap population vs. energy



Mulliken overlap population:

 $\Psi = c_1 \chi_1 + c_2 \chi_2$ ED (MO): $1 = c_1^2 + c_2^2 + 2c_1 c_2 S_{12}$



 $a = d_{H-H}$

2 H per unit cell, thus 2 χ_j orbitals : χ_j^{σ} and $\chi_j^{\sigma^*}$ $\sim \chi^{\sigma^*}$ (vacant)

 \bigcirc \bigcirc \checkmark χ^{σ} (occupied)

Bloch orbitals in the irreducible part of the first Brillouin zone $(0 \le k \le \pi/a')$: $\phi(k) \propto \sum_{i} [\exp(ikR_i) \cdot \chi_i]$, with $R_i = j.a$ and $\chi_j = \chi_j^{\sigma}$ or $\chi_j^{\sigma^*}$

 $k = 0 \qquad \varphi_{\sigma}(0) \ \alpha \sum_{j} \exp(0) \ \chi_{j}^{\sigma} \text{ and } \varphi_{\sigma} * (0) \ \alpha \sum_{j} \exp(0) \ \chi_{j}^{\sigma}^{*}$ $\varphi_{\sigma}(0) = 0 \qquad \varphi_{\sigma}(0) \ \alpha \sum_{j} \exp(0) \ \chi_{j}^{\sigma} \text{ and } \varphi_{\sigma} * (0) \ \alpha \sum_{j} \exp(0) \ \chi_{j}^{\sigma}^{*}$ $k = \pi/a' = \pi/2a \qquad \varphi_{\sigma}(\pi/a') \ \alpha \sum_{j} \exp(i\pi j) \ \chi_{j}^{\sigma} \text{ and } \varphi_{\sigma} * (\pi/a') \ \alpha \sum_{j} \exp(i\pi j) \ \chi_{j}^{\sigma}^{*}$



Band structure: E(k) = f(k)





Exercise: Generate the band structure of the double cell (a' = 2a) by considering the interaction between two interpenetrating H networks



The **H** metwork generates a flat band (very weak overlap between red atoms). Similarly, the **H** metwork generates a flat band. Both flat bands interact and repel each other. They interact strongly at k = 0 (S is maximum) They do no interact by symmetry at $k = \pi/a'$ (S = 0)



The Peierls Distortion

Peierls distortion (solid) \equiv Jahn-Teller distortion (molecule)



Metal, Semi-conductor or Insulator?



 ρ (resistivity) inversely proportional to DOS n(e) at ϵ_{F}

Exercise: Generate the π -type band structure of a regular zigzag polyacetylene chain and show that it is Peierls unstable



NB. The Peierls distortion is not associated with a doubling of the unit cell, but with the loss of an helicoidal axis.

Exercise: Generate the band structure of a chain made of equidistant carbon atoms

---=C=C=C=C---

- a) Each carbon AO generates a Bloch function.
- b) The σ -type 2s and $2p_z$ Bloch bands interact and strongly repel each other. The resulting σ and σ * bands have no particular shapes and are rather flat.
- c) The π band is doubly degenerate.



A chain of equidistant carbon atoms... Peierls Instability...



- a) Identify the repeat unit (unit cell)
- b) Consider the content of the repeat unit as a molecular fragment
- c) Consider the MO diagram of this repeat unit (AOs if single atom). It contains the interactions between all the AOs inside one individual unit cell
- d) Generate the Bloch functions developed on each MO. A Bloch function developed on one particular χ MO of the unit cell describes the interactions between all the χ_j MOs of different unit cells
- e) Allow the Bloch orbitals to interact at each *k*-point if symmetry allows it to get the crystal orbitals (CO). The final band structure will arise

More Dimensions

Two Dimensions



Bloch orbitals: $\phi(k) \propto \sum_{j_1} \sum_{j_2} \exp(ikR_j) \chi_j$ wh

where $k = (k_x, k_y)$ $R_j = j_1 a_1 + j_2 a_2 (j_1 \text{ and } j_2 = 0, \infty)$ $\chi_j = 1s(H)$

Two dimensions

Square lattice of H atoms (1 H per unit cell)



Bloch orbitals:



Two dimensions









Bulk fcc Al



Bulk fcc Ni

More examples: graphite and diamond





Construction of BOs, $\phi(k) \implies$ Band structure, E(k) = f(k)





Band structures and Fermi surfaces







Fermi surfaces explain the dimensionality of metallic properties



1D metal (along a axis)



pseudo-1D metal (along a axis)





2D (anisotropic) metal (along a+b and a-b axis)

2D (isotropic) metal (close loop)

Fermi surfaces explain distortions in solids due to electronic instabilities

(concept of Fermi surface nesting)









A Fermi surface is nested by a vector q when a section can be moved by such a vector to be superimposed on another section

A metallic system with a *nested* Fermi surface is subject to a metal-insulator phase transition (interaction of occupied and vacant crystal orbitals of same symmetry of $E = E(k_F)$ related by q)



• Quantum chemical methods

There are different ways to solve the Schrödinger equation for periodic systems

Crystal orbitals $\Psi_n(k)$



Crystal orbitals $\Psi_n(k) \Longrightarrow e_n(k)$ for specific *k* wave vectors along symmetry lines of the irreducible part of Brillouin zone

Band dispersion relation, E(k) = f(k)



Brillouin zone



one band



two bands with different symmetry



two bands with identical symmetry





• Fermi surface

 $e_n(k_i) \implies e_n(k_F) = e_F$ (constant energy surface)



The Theoretical Machinery at Work...

Application to the Texbook Example of TTF-TCNQ, the First Organic Molecular Metal



Figure 2. Arrangements found for neighboring molecules in stacks of (a) TCNQ, (b) TTF in TTF-TCNQ crystals.

TTF-TCNQ: A truly organic molecular metal $\sigma \sim 5.10^3 \Omega^{-1}$.cm⁻¹ at 80 K (-210 °C) (10⁶ for Cu) What's special for TTF-TCNQ?

The Texbook Example of TTF-TCNQ



The Texbook Example of TTF-TCNQ



The Texbook Example of TTF-TCNQ

TCNQ salts are generally semi-conductors

Na(TCNQ) \longrightarrow (TCNQ₂)²⁻ dimers

 $K(TCNQ) \longrightarrow (TCNQ_2)^{2-}$ dimers

 $Cs_2(TCNQ) \longrightarrow (TCNQ_3)^{2-}$ trimers

 $NEt_4(TCNQ)_2 \implies (TCNQ_4)^-$ tetramers

TTF salts as well...

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What's special for TTF-TCNQ?

DFT band structure (two TCNQ and two TTF molecules per unit cell):



FIG. 2. DFT band structure near the Fermi level along the three major high-symmetry lines of the Brillouin zone of TTF-TCNQ.



FIG. 3. Theoretical band dispersions along ΓZ showing the molecular origin of the bands. The size of the symbols represents the charge of each state residing on the TCNQ (left panel) and TTF (right panel) molecules.

The TTF and TCNQ bands overlap at the Fermi level (1D metal)

M. Sing, Phys. Rev. B. 2003, 68, 125111

The Electronic Structure of TTF-TCNQ

What's special for TTF-TCNQ?

The TTF and TCNQ bands overlap at the Fermi level



No distortion (at room temperature)

TTF-TCNQ is a 1D metal

Examples of Molecular Materials and their Electronic Structure

The superconductor κ -(ET)₂Cu(NCS)₂ (T_c = 11 K)



... An extensive work to shift the Fermi level by replacing part of Cu(I) with Cu(II) has so far been unsuccessful for κ -(ET), Cu(NCS), however, successful in a very limited range in κ -(ET)₂(Cu^I₂. $_{x-v}Cu^{II}_{x}$ {(CN)_{3-2v}[N(CN)₂]_v} [24]. A uni-axial strain of 1 kbar applied along the c-axis of κ - $(ET)_{2}Cu(NCS)_{2}$ increases T_{c} by 1 K [25] owing to the much flattened 1D like Fermi surface under pressure as expected from the band calculation giving rise to an enhancement of electron correlation. A uni-axial strain along the b-axis monotonically decreases T_c , probably due to the increase of the bandwidth and hence the decrease of $D(\varepsilon_{\rm F})$.





Calculated Fermi surface (top), energy dispersion (bottom left) and density of states $D(\varepsilon)$ (bottom right of the organic network of κ -(ET)₂Cu(NCS)₂

Electronic Structure of the Superconductor κ -(ET)₂Cu(NCS)₂



states $D(\varepsilon)$ (bottom right) of the organic network of κ -(ET)₂Cu(NCS)₂

Electronic Structure of some (BEDO-TTF) Complexes

Calculated band structures, density of states (DOS), and Fermi surface of (d) (BEDO-TTF)₅(HCP)(PhCN)_{0.2}. The nesting vector **q** is represented by arrows in d.

Two Fermi surfaces superposed to each other by translation vector $\mathbf{q} = 3/5 \ b^*(\mathbf{q} = \mathbf{k} - \mathbf{k}' \ (\mathbf{k} = 0.3 \ \pi/b))$ (arrows in Figure). This nesting of the Fermi surface (often seen in the quasi-1D materials) typically causes a CDW- or SDW-associated metal-insulator transition.

Electronic Structure of some (BEDO-TTF) Complexes

Calculated band structures, densities of states (DOS), and Fermi surfaces of (a) $(BEDO-TTF)_{10}(CF)_4(H_2O)_3$, (b) $(BEDO-TTF)_5(HCTMM)(PhCN)_2$, and (c) $(BEDO-TTF)_4(SQA)(H_2O)_6$.

Home Work: What is the Band Structure of (EDO-TTF)₂PF₆?

Fig. 1. (A) Schematic views of the lattice and electronic structural changes accompanying the M-1 phase transition in [EDO-TTF], PL, A side view of an EDO-TTF molecule is shown. The unit cell includes two and four EDO-TTF] PL, A side view of an EDO-TTF molecule is shown. The unit cell includes two and four EDO-TTF] redictules in M and I phases, respectively (15). In the I phase, holes are localized on EDO-TTF molecules with a flat structure due to CO, and quasi-mentral molecules show a bent structure. In the M phase, charges (holes) are delocalized and PL, [acceptor] molecules estilling disorder (15–18). [B) Schematics for the energy change accompanying M-1 transition and the structure of the EDO-TTF molecules.

Science 2005, 307, 86-89

Home Work: What is the Band Structure of (EDO-TTF)₂PF₆?

Expected band structure (four EDO-TTF molecules per unit cell):

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